3.1
Form finding

During the twentieth century, architects and engineers both developed ways of designing complex structural forms by experimenting with physical models and through borrowing from structures found in nature. Finding and creating new structural forms was accomplished by extracting geometric information from physical models, in particular three-dimensional compressive surfaces—shells—or three-dimensional tensile surfaces—membranes. With the advent of computer-aided design along with an increased knowledge of the behavior of materials, a variety of approaches to form finding can now be pursued using computer programs to calculate optimum structural solutions for given geometric parameters.

Suspension models

A "catenary" curve is derived from hanging a chain or a cable that, when supported at each end, is allowed to bend under its own weight. In the case of a suspension bridge, the cables that are stretched between the masts form a catenary curve; however, once the cables become loaded (by hanging a deck from vertical cables placed at regular intervals) the curve becomes almost parabolic. When a catenary curve is inverted, it forms a naturally stable arch. Arches produced in this way are structurally efficient, because the thrust into the ground will always follow the line of the arch.

To generate compressive, shell-like structures, a net or fabric is suspended from a set of points and then fixed in position by saturating it with plaster and/or glue. This is then flipped over (mirrored horizontally) to create a thin shell-like form. Owing to their structural efficiency, these forms may be described as "minimal surfaces."

Soap films

Soap bubbles (see section 1.4) are physical illustrations of a minimal surface. A minimal surface is more properly described as a surface with equal pressure on the inside and the outside. A film obtained by dipping a wire frame contoured closed shape into a soapy solution will produce a minimal surface.

Structural application

Designers such as Frei Otto and Heinz Isler have used form-finding structural prototypes as both design and engineering tools. In the case of Otto—and specifically his work with soap films—these models were painstakingly photographed, logged, mapped, and drawn, generating profiles for latterly realized projects. Heinz Isler, whose interest was in optimally engineered thin reinforced concrete shells, regularly used physical scale models to generate surface geometries. These reverse-engineered plaster models were very accurately measured on a custom rig, with the subsequently plotted profiles used as the basis for his large-scale "catenary" shells.

Virtual form finding

Form-finding software is now widely available as a design and analysis tool and is no longer solely the domain of the professional engineering office. Form-finding software is based on principles such as the geometric optimization of the soap-film modeling techniques pioneered by Frei Otto. Typical form-finding software contains a range of procedural geometric transformations as well as ascribable properties for the constituent material construction and arrangement, which may include fabric type, steel cabling, and connectors. The virtual model can then be subject to prestress and live load simulations. While there is no question of the value of these excellent new tools, which allow for fast iterative modeling, there are still good arguments for physical prototyping. The physical scale model as an analog of the final physical construction has much to tell the designer, not least in relation to material behavior and project-specific constructional and assembly issues.

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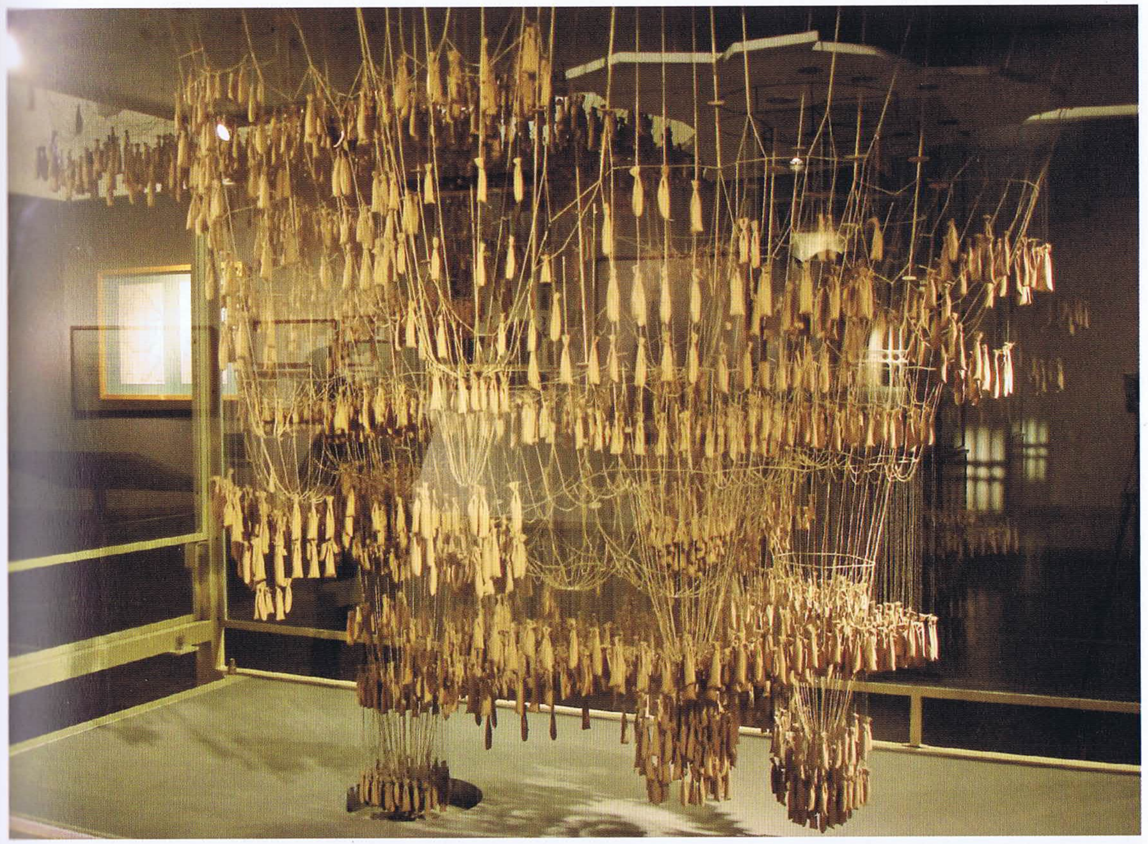

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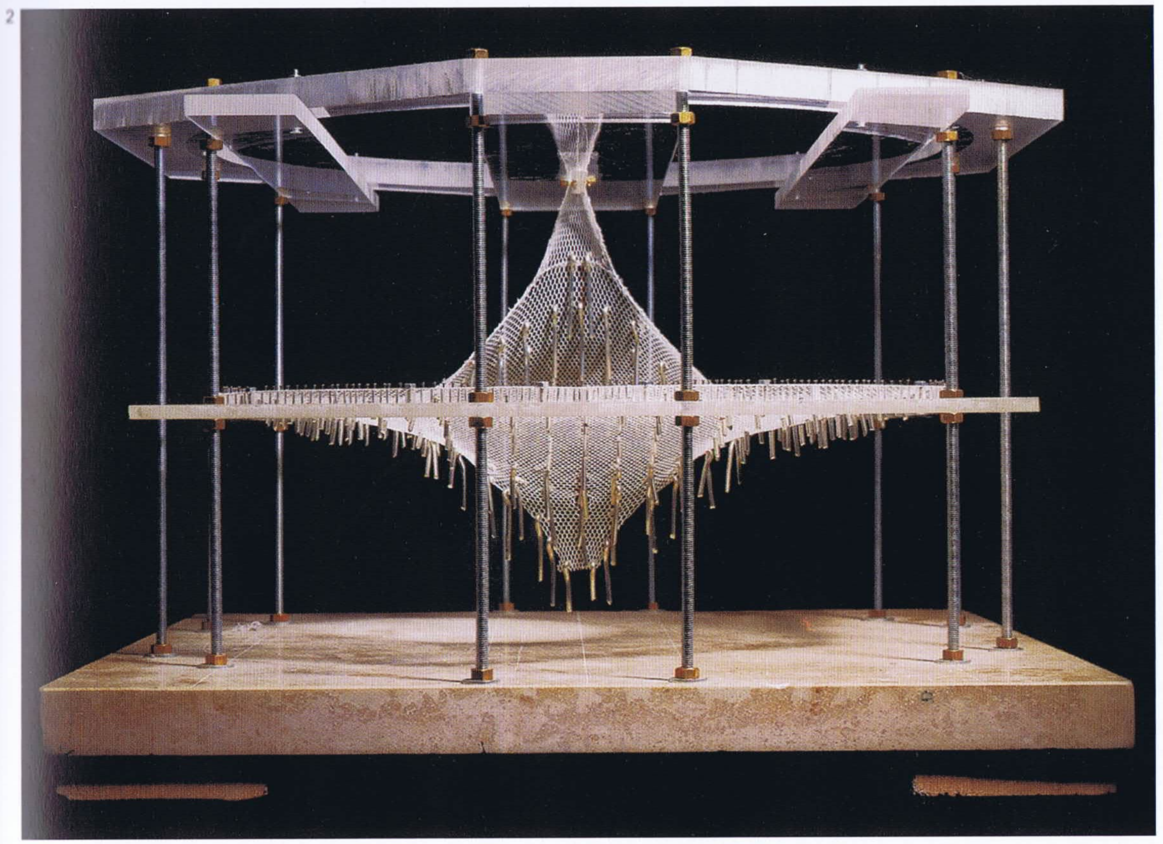
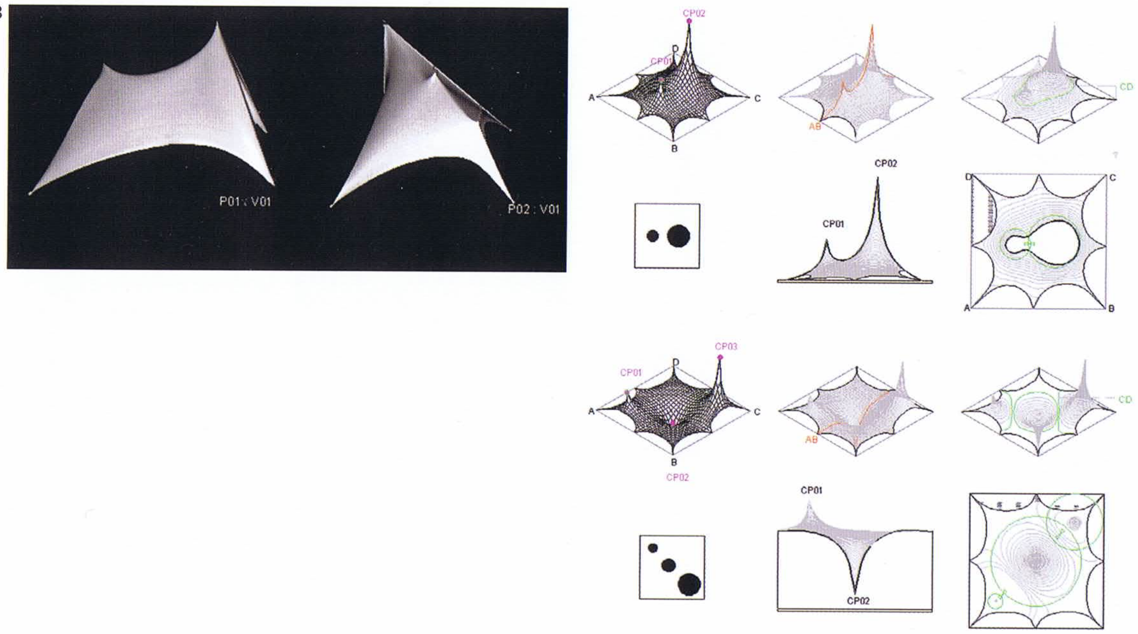


1 Hanging nets
Antoni Gaudí's models explored the design of vaulted compression structures using the same principle as the catenary curve, by hanging weights from flexible nets and then inverting the resultant forms

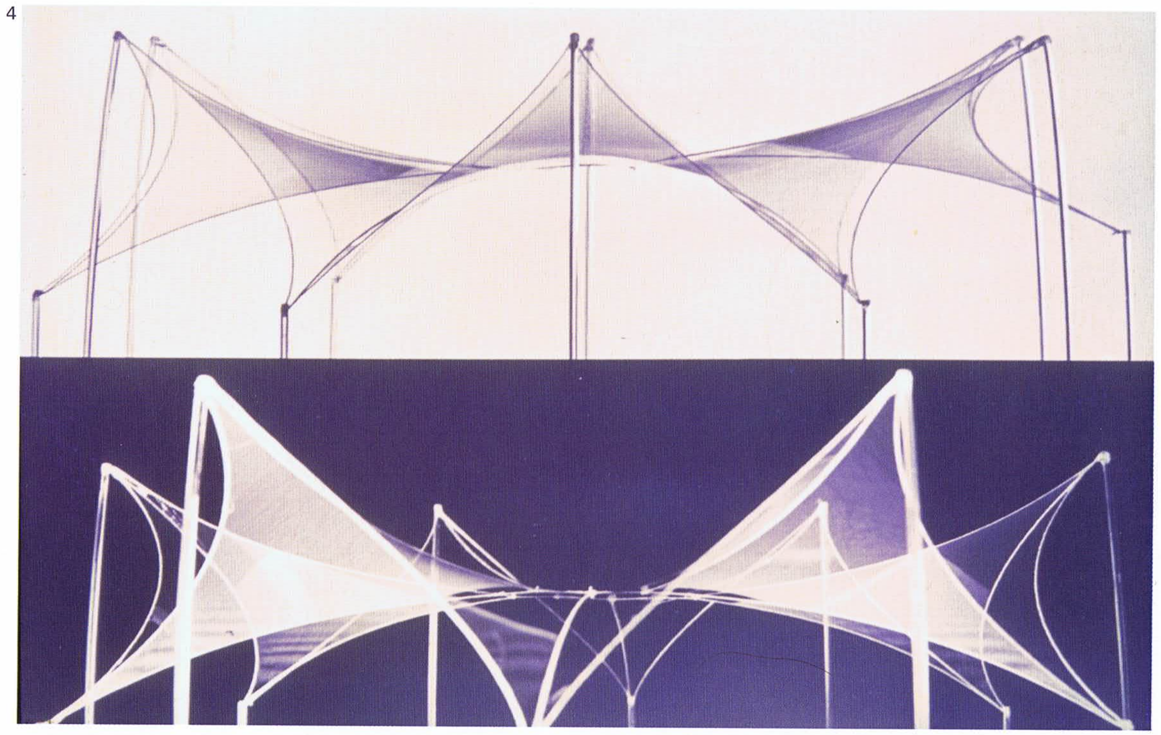
2 Suspension model
Structural model made to establish the form of the arches for a new train station in Stuttgart, Germany, 2000, by Christoph Ingenhoven and Partner, Frei Otto, Büro Happold, and Leonhardt and Andrae

3



3 Control points
Images created using form-finding software for the design of membrane structures. Control points (CP) are used to create space. The program operates in such a way that when a force is applied to one point the load of the force is distributed homogeneously so that the membrane is always under tension to produce a smooth transition between points.

4

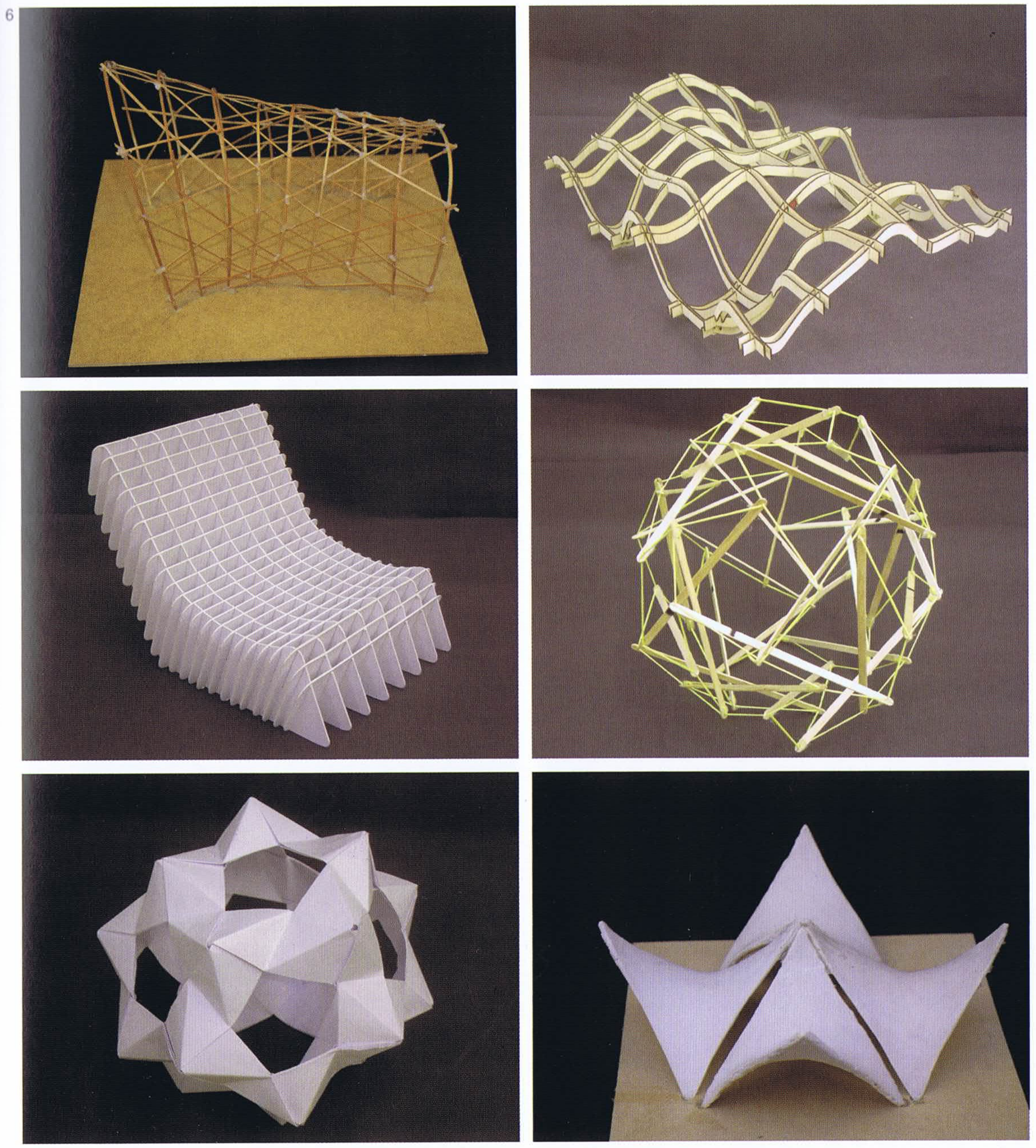


4 Soap-film model
Model by Frei Otto for the design of a membrane structure using soap film on a wire-bounded framework. This is both a minimal and an anticlastic surface, which can be graphically described as a "double-ruled" surface, i.e. one that can be described using a grid of straight lines.

5

5 Ice shells
Heinz Isler designed a technique whereby fabric was draped over masts and then saturated with water. In freezing temperatures the membranes solidified and the masts could be removed, forming "ice shells." Shown here is an image of ice shells constructed at Cornell University, Ithaca, NY, in 1999 by Dr. Mark Valenzuela and Dr. Sanjay Arwade, with the assistance of undergraduates from Dr. Valenzuela's Modern Structures class.

6



6 Modeling techniques
Structural models can employ a range of form-finding techniques according to the properties of the materials used, as shown in the examples here. All models by second-year undergraduate students at the School of Architecture, University of Westminster, London, 2007–9.

Left to right, top to bottom:

Gridshell vault, formed using (elastic) timber strips that are held in tension and fixed at the base of the model.

Complex surface built up of laser-cut profiles in an interlocking grid

Interlocking cardboard profiles used to model a formwork core

Disposable sticks and elastic bands employed to model a collapsible tensegrity dome

Paper ribbons folded and interlocked to generate a regular solid

Hyperbolic parabolas (saddle shapes) generated by saturating a hung fabric with plaster

3.2
Load testing

Load testing has always been a critical part of structural design development. While the prediction of the behavior of materials and construction elements may be calculated mathematically and with computer models (such as Finite Element Analysis and Computational Fluid Dynamics, see pages 106–9), much can be learned by prototyping and observation. The first time it was understood that reinforced concrete could flex and bend under load was on the completion of Berthold Lubetkin's Penguin Pool at London Zoo in the 1930s.

As can be seen from Robert Stevenson's work on the Bell Rock Lighthouse, there is evidence that the use of prototype models was paramount to the resolution of successful structural design to resist the enormous power of the sea. Similarly, monolithic, compressive vaults and domes have, from Gothic times, required innovative construction techniques and materials that are still under constant development. This section is also illustrated by a set of practical, problem-solving exercises, showing examples of a considerable variety of resolutions.

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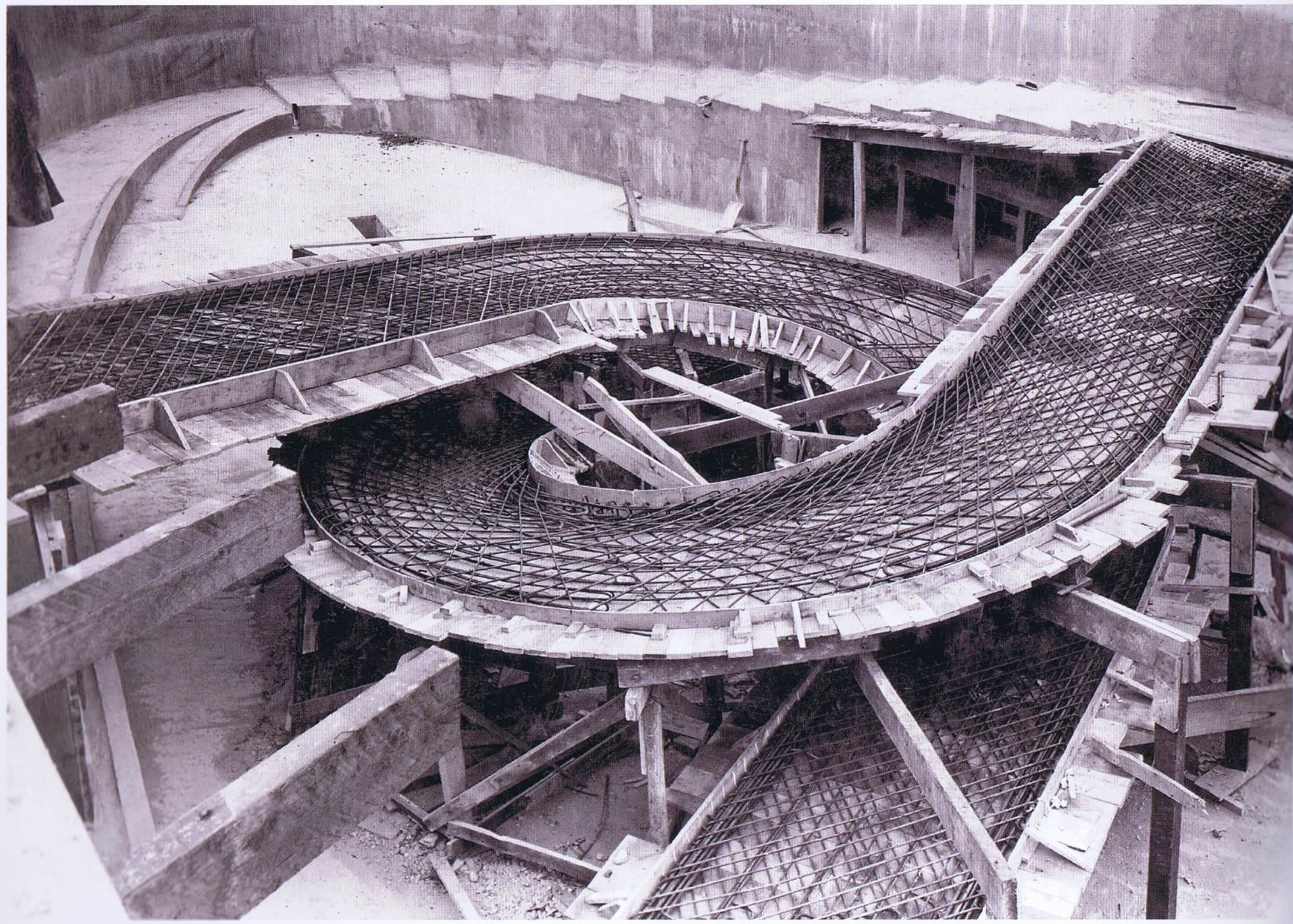


1
The spiraling, reinforced-concrete ramp of Lubetkin's Penguin Pool at London Zoo, under construction in 1933

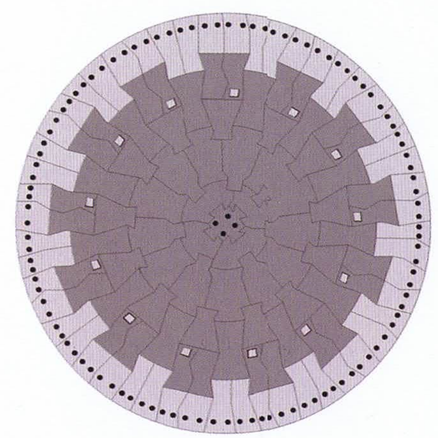
2

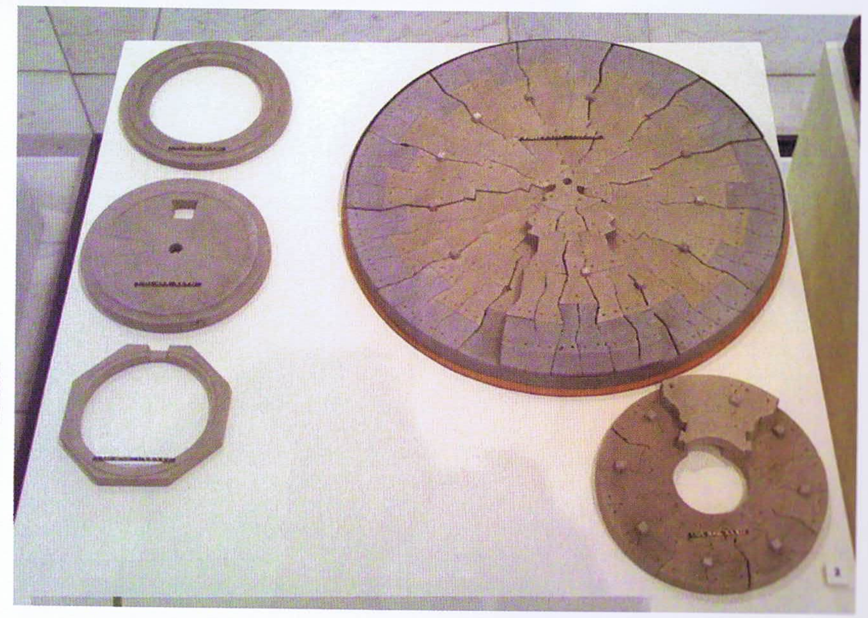

2 Bell Rock Lighthouse
The design of the Bell Rock was the culmination of knowledge gained from the construction of previous lighthouses (many of which had failed) and from prototyping with scale models. John Smeaton had built the Eddystone Lighthouse in 1759, pioneering the use of stone. Not only were the stones "dovetailed" to interlock with each other, but they also employed wedges similar to the dowels in a "scarf" joint. The ideal profile to resist the enormous impact from wind and waves was found to be parabolic in shape; Robert Stevenson and John Rennie are known to have built scale models against which they would throw buckets of water.

Left:
Photograph of Bell Rock Lighthouse showing the parabolic curve at the base

Below left:
Section through the interlocking stone blocks at foundation level

Below:
Models of the construction details. Held at the Museum of Scotland, Edinburgh





3

3 Thin-shell monolithic domes
Modern (lightweight) materials technology linked to the use of air-supported formworks has greatly improved the efficiency and practicality of casting concrete domes (which are similar in shape and structure to an eggshell). Inspired by prototypes developed by Félix Candela, Pier Luigi Nervi, and Anton Tedesko among others, shown here is a project by Dr. Arnold Wilson at the Brigham Young University Laboratories, Idaho, USA, to load test a thin-shell concrete dome. Using air-supported form technology (made from nylon-reinforced vinyl, which is left in place as a watertight finish), the dome is formed using polyurethane foam and sprayed (reinforced) concrete.

Above:
Inflatable formworks, showing reinforcing

Left:
Load testing a dome

4

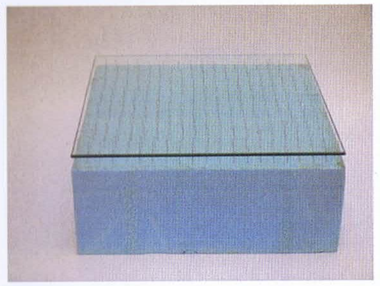
4 Brick vaults
Inspired by the work of Eladio Dieste and others, the Vault201 prototype vault was built by MIT architecture students at the Cooper-Hewitt National Design Museum, New York. The vault spans 16ft, is 1½in thick, and uses 720 bricks. The curvature of the vault is composed of splines that vary in profile but are fixed in length in order to keep an equal coursing pattern and to save custom-cutting too many bricks. In the end, as a result of prototyping, a taxonomic system of three different brick modules was developed.

To quote the students: "1) learn from building, 2) analyze and abstract as rules, and 3) re-embed into the design process."

(See http://vaulting.wordpress.com/ for a full account of this project.)

The following illustrations are taken from first-year undergraduate student projects conducted at the University of Westminster, London, UK, from 2009 to 2011. Students were introduced to common construction materials, fabrication processes, and workshop practices and were then asked to design and build a 1:1 scale object in order to solve a specific structural problem. Prototyping took the form of sketching, modelmaking, and experimenting with materials, and students learned how the act of "making" can form an integral part of the design process. Objects were assessed according to structural efficiency (lightness), craftsmanship, construction details, and the innovative use of materials.

5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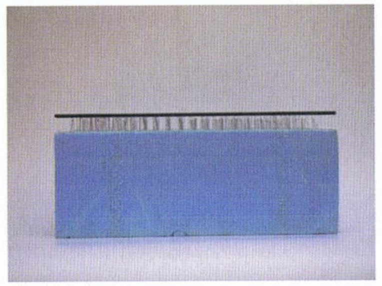
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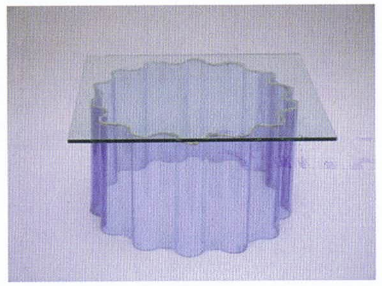
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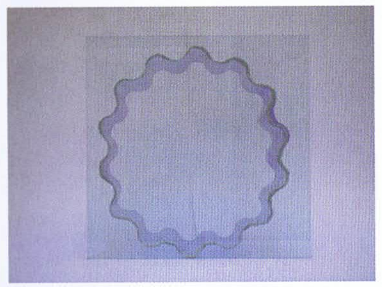
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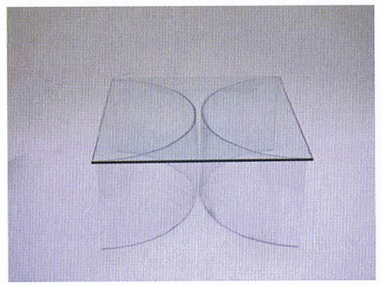
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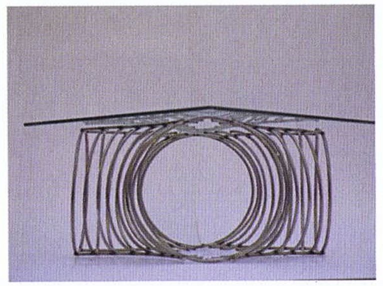
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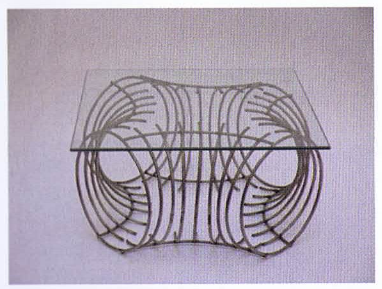
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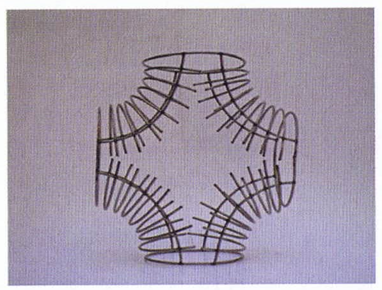
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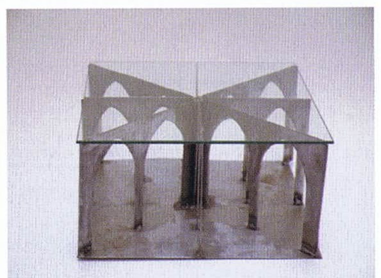
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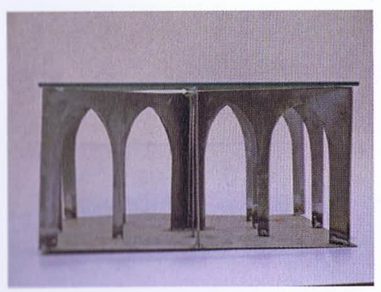
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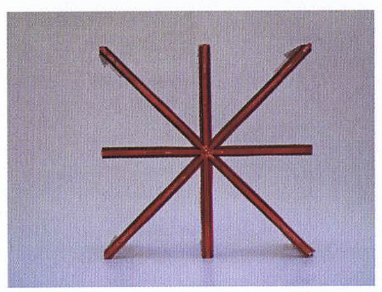
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5 Supports for a sheet of glass
In this project the students explored testing methods to support a human body 8in in the air on a 0.62 sq in, 0.25in-deep sheet of (untempered) glass. All examples shown employ elements that are primarily in compression. (See section 2.1.5.2 Axial compression.)

a, b
Multiple, point-loaded structure exploring iteration and scale

c–e
Stiffness achieved using corrugation and stability through the use of a circular plan

f–h
The arch and the cantilever principle combined

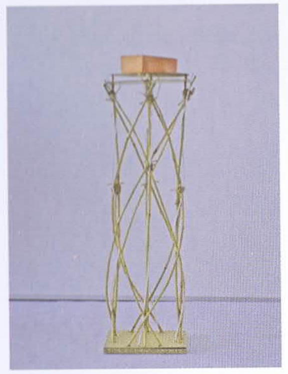
i, j
Pointed arches used as colonnades

k, l
A pointed arch, perforated for lightness, acting as a portal frame


m

m
Students testing their support structures

6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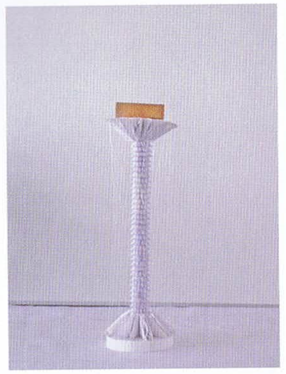
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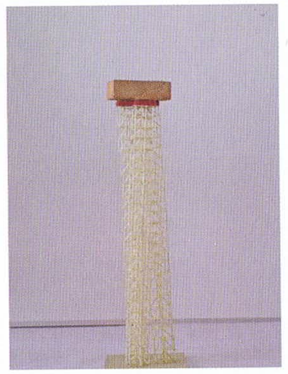
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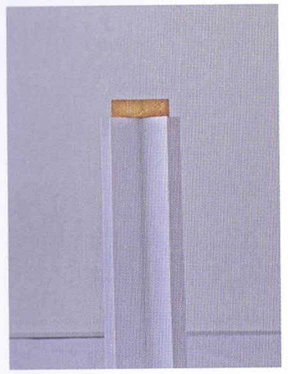
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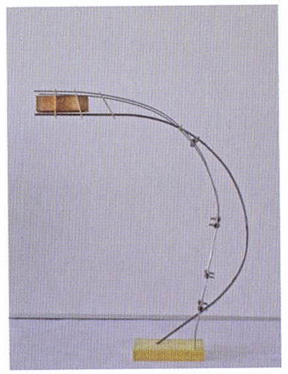
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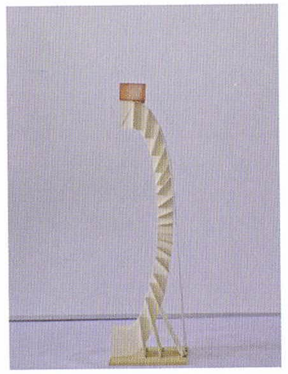
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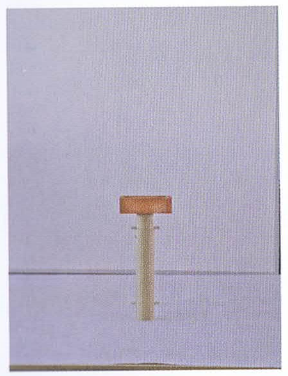
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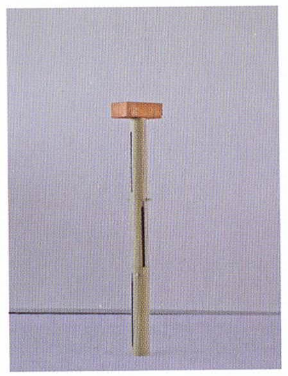
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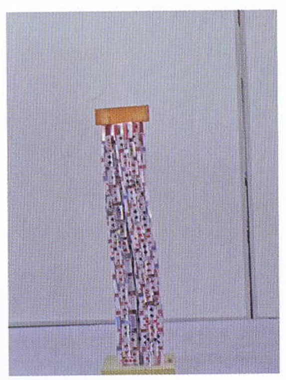
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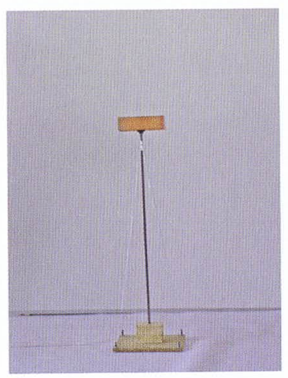
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6 Brick-supporting plinth
The brief was to design a column that could support a single Fletton brick at a height of 3 feet, without bending, buckling, or rotating. The load was considered primarily to be vertical, though the plinth should resist torsion (see section 2.1.4.1 Stress: Element under torsion). Maximum footprint was set at 10 x 10in. These efficient minimal structures were to be designed to fail under the load of two bricks.

a
This project set out to explore the structural potential of the double helix by employing elements made from a stiff material with the capacity for elastic deformation—in this case, bamboo. Torsive forces were applied in order to twist opposing elements in opposite directions; they were then locked at either end so that the forces canceled each other out. This produced an extremely rigid structure with a high strength-to-weight ratio

b
This project consisted of a mast that was made up of multiple, folded (paper) elements slotted around a cylindrical core. Rigidity was achieved through a system of bracing that would resist torsional movement by tensioning lever arms at the top and base, using a network of triangulating wires

c
A lightweight, compressive lattice consisting of three masts that were intertwined for stability

d
A monolithic, planar structure whose form was derived by extruding from a simple plan. A series of ribs was connected (critically) at the point of rotation. To prevent the thin, planar ribs from buckling under load, they were individually laminated (using foamboard)

e
This project explored the possibility of cantilevering the brick, while at the same time employing a minimum number of primary elements. By using two rods with the capacity for elastic deformation they could be "laminated" together to act simultaneously in tension and compression to form a rigid structure

f
A single, tapering lever arm was stiffened using a series of ribs, which also acted to stabilize the structure at ground level. The vertical cantilever was completed by tensioning the lever arm back to the base of the structure

g–i
This deployable solution used a telescoping mechanism. A set of cardboard cylinders was slotted so that they could be pegged at various heights

j
This project set out to leave a clear space below the brick while also being deployable. The solution involved using three armatures that were each centrally hinged. The desired height was achieved by tensioning each of the arms to its neighbor with the appropriate length of cable

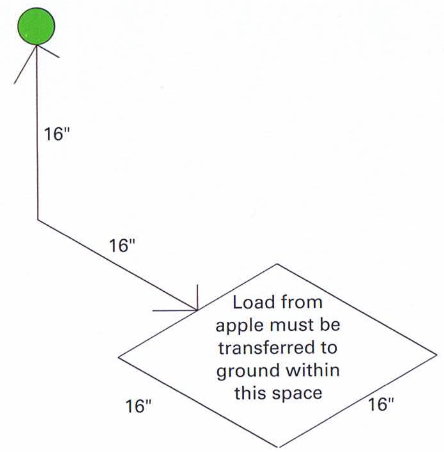
k
The core of the mast consisted of cards that were stacked and slotted together vertically. Rigidity was achieved by tensioning the top to the base

l
This simple column was stabilized by tensioning cables to the base plate


m

m
Supported by 200 helium balloons, the brick was held by a perforated, polystyrene beam in order to stabilize and spread the load

7



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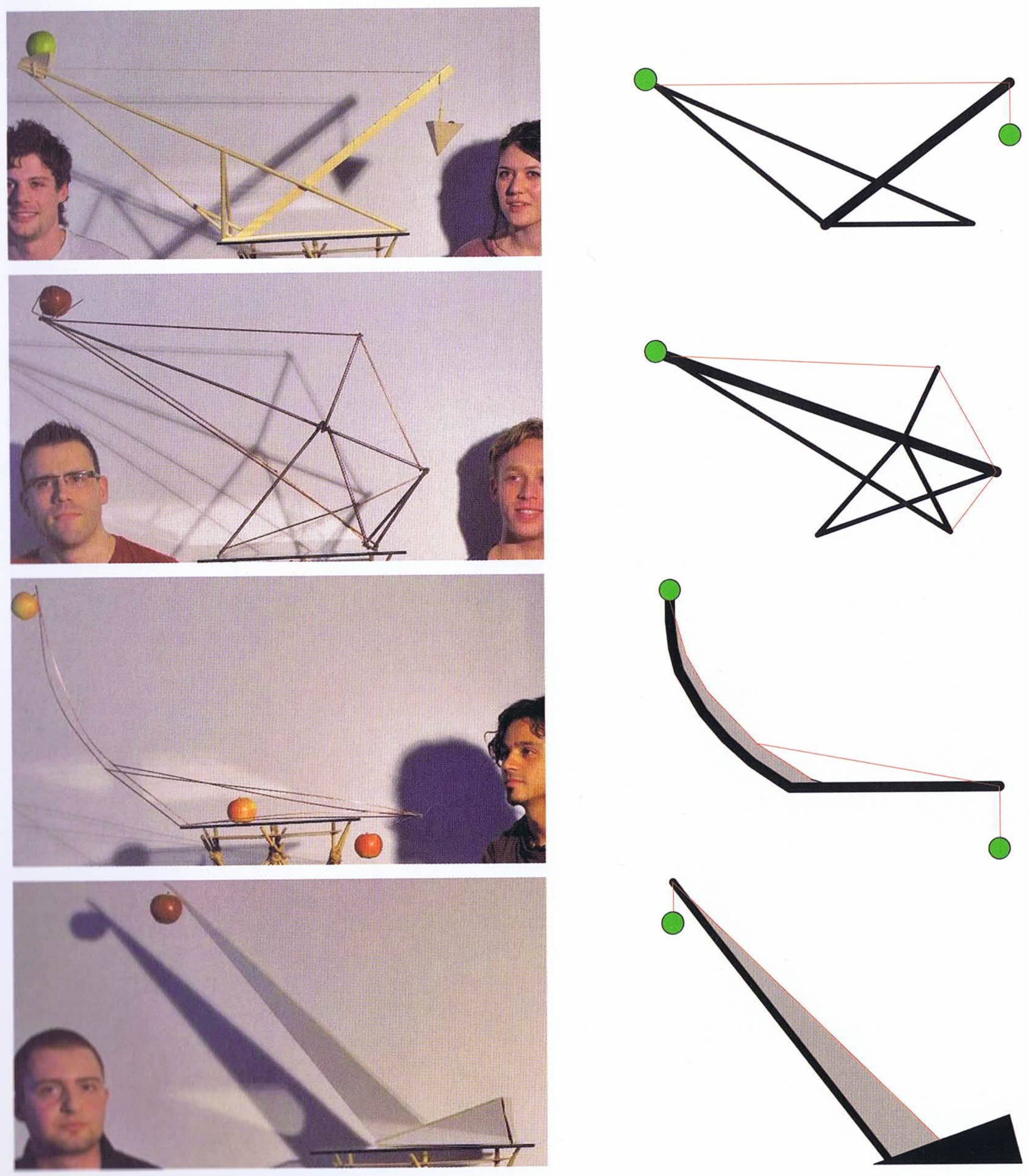


b

7 Cantilever support for an apple
The following illustrations show the results of an exercise to explore solutions to cantilevering an apple 16in horizontally and 16in vertically from a 16 x 16in footprint. The load was considered primarily to be vertical, though the apple should remain stable in the horizontal plane. The diagrams describe the tensile (red) and compressive (black) elements at work in the structures.

a
Diagram explaining the general requirements for each structure

b
Photographs of four selected structures with diagrams describing the tensile (red) and compressive (black) elements

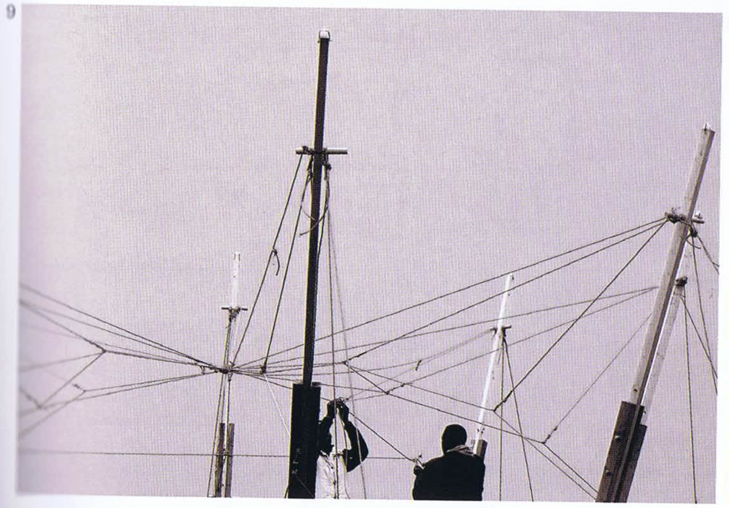
c
The students' solutions to this structural problem were varied and inventive

c

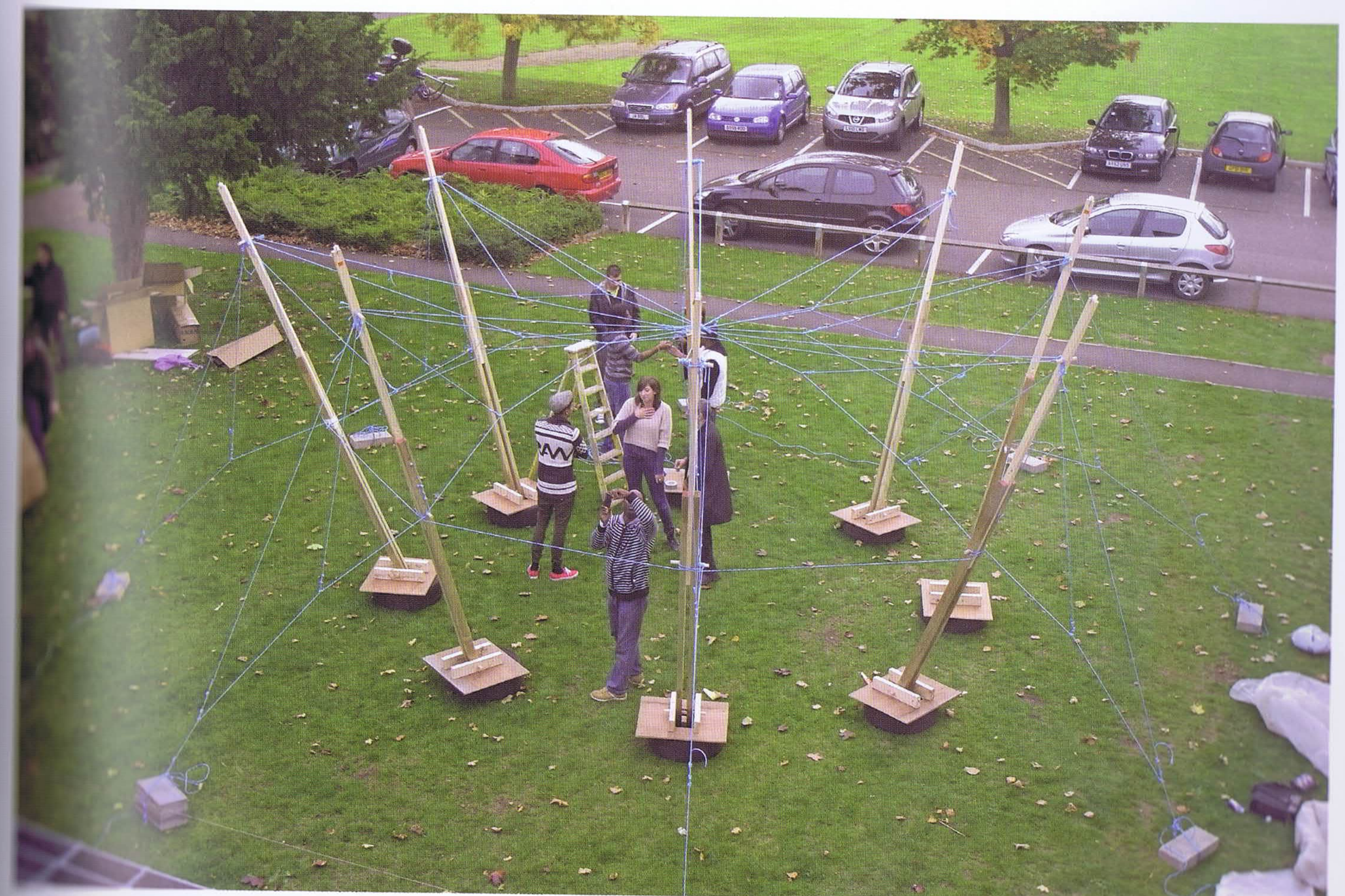
8

8 Glass "sandwich" panel spanning element
This structural prototype developed by David Charlton at the University of Westminster, London, UK, creates a "sandwich" panel using traditional incandescent light bulbs close-packed in hexagonal plan formation and bonded to thin sheets of glass using structural silicone. The glass honeycomb-like core created from recycled light bulbs utilizes the relative longitudinal compressive strength of the bulb similar to that of an eggshell (see Section 1.3 Eggshell). The close packing of the bulbs resists the tendency of the bulbs to buckle (and fracture), providing lateral stability. This novel prototype reminds us of the usefulness of putting distance between the top and bottom chord of a beam, truss, or spaceframe, thus creating structural "depth" with which to "span." This prototype also shows how, with thoughtful geometric configuration, compressive strength can be maintained with lightweight and even fragile materials maintaining impressive strength and reducing dead (static) loads.

9



9 Cable net structure
A cable net structure for a DIY version of London's O_2 Arena (formerly the Millennium Dome) was constructed by first-year undergraduate students at the University of Greenwich, London, UK. This 1/36 scale model utilized all of the structural attributes of the original, albeit simplified by using eight rather than twelve uprights (compression members) for this mast-supported cable net.



3.3
Visualizing forces

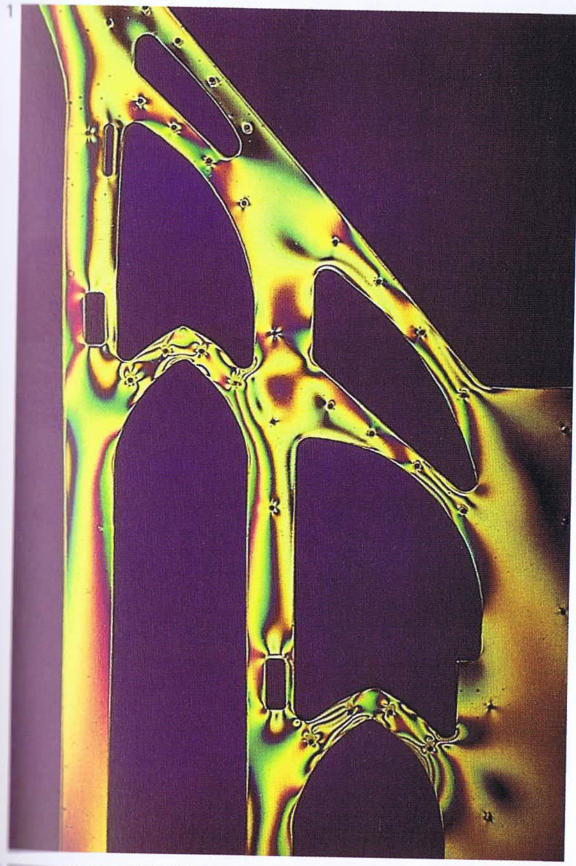
A key development in engineering analysis has been the ability to visualize forces within a "structural model." In a process developed at the beginning of the twentieth century, photoelastic modeling allowed scale models fabricated from transparent cast resin to have the internal structural forces made visible. Using two polarizing lenses set each side of a scale model, light is passed through the rig, and birefringence (double refraction) occurs in direct relation to localized stress patterns. Whereas physical models may have been previously used to verify structural calculations, these photoelastic structural "analogs" allow the designer to simultaneously test and observe structural forces and structures in action.

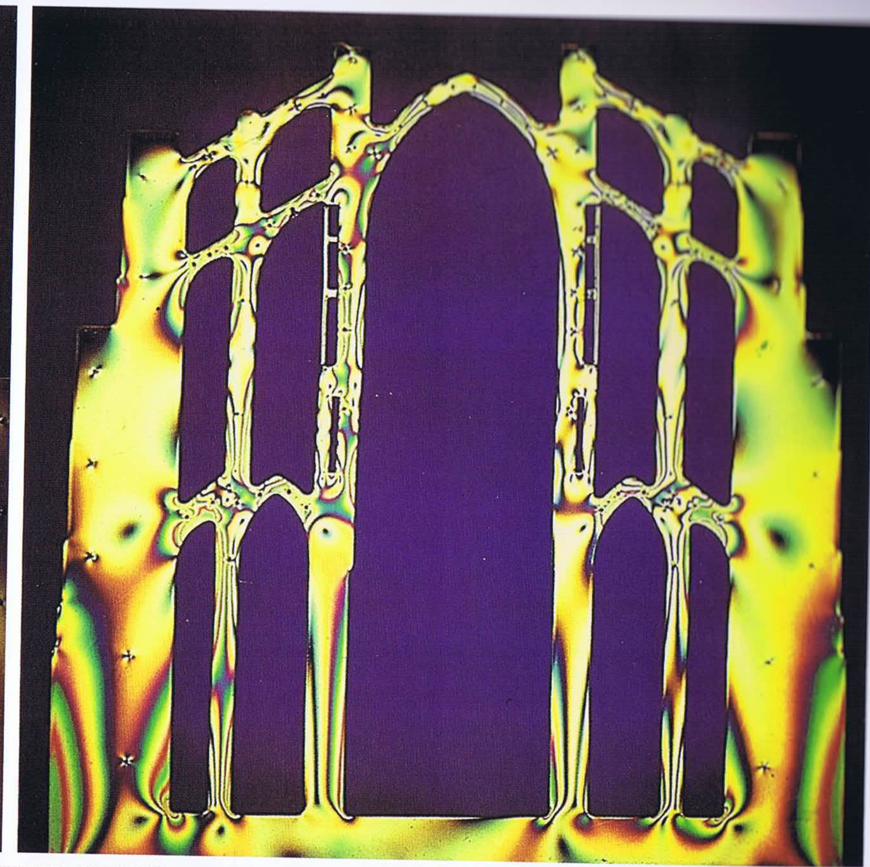
With the development of Finite Element Analysis (FEA) and application of the Finite Element Method (FEM), graphical computing allows the designer to model a two- or three-dimensional structural system or connection and study the fourth dimensional effects of gravity, static and live loads, and other applied structural forces. The advent of inexpensive computing allows a fully integrated Building Information Model (BIM) to be recast or reconfigured with information feedback from FEA analysis and additional dynamic environmental factors such as wind loads, modeled with Computational Fluid Dynamic (CFD) software.

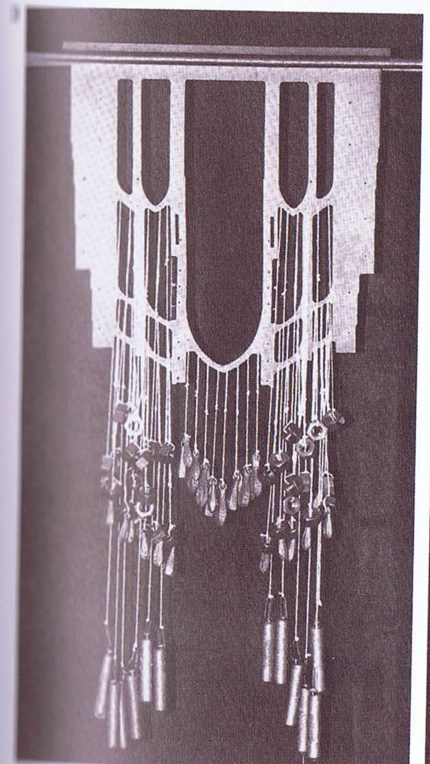
Photoelastic modeling

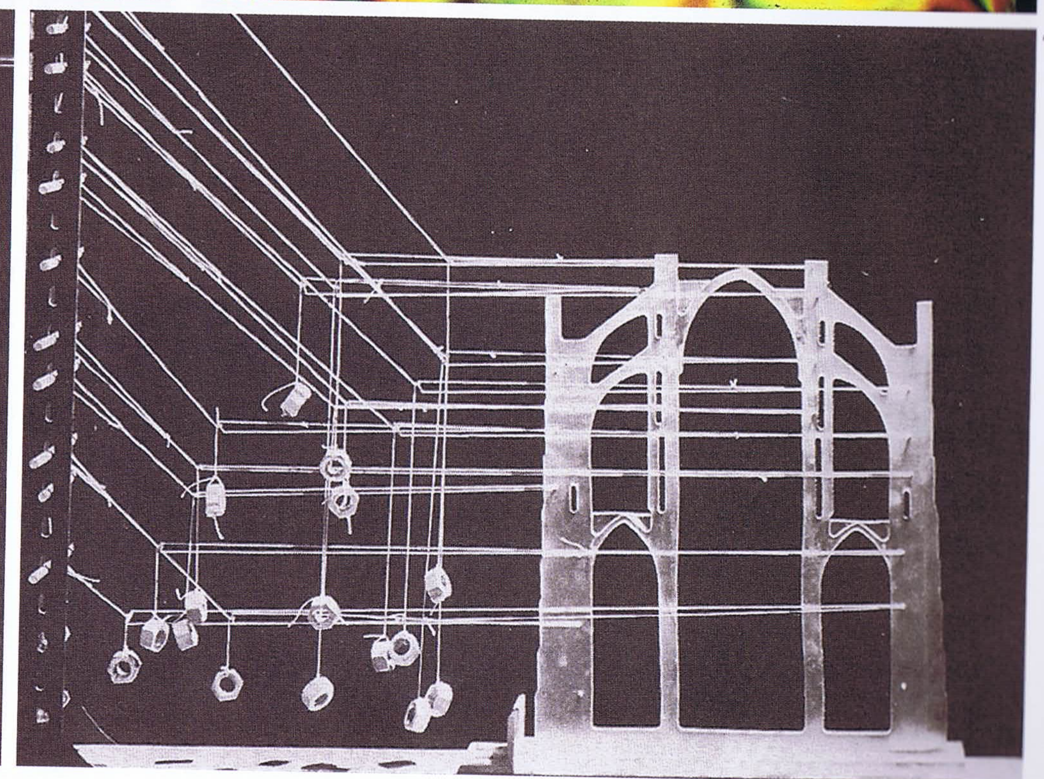
Photoelastic modeling is an experimental method to determine stress distribution in a material, and is often used for determining stress-concentration factors in complex geometrical shapes. The method is based on the property of birefringence, which is exhibited by certain transparent materials. A ray of light passing through a birefringent material experiences two refractive indices. Photoelastic materials exhibit this property only on the application of stress, and the magnitude of the refractive indices at each point in the material is directly related to the state of stress at that point. A model made out of such materials produces an optical pattern representing its internal stress patterns.

Professor Robert Mark of Princeton University brilliantly illustrates both the method and analytical usefulness of the photoelastic technique in his book *Experiments in Gothic Structure* (MIT Press, Cambridge, MA, 1982), where a series of comparative (sectional) models of some of the great Gothic cathedrals of Europe are photoelastically modeled and subjected to notional live (wind) loads. These live and responsive illustrations of stress patterns in a given structure provide valuable indicative evidence of localized "hot spots" for study or amelioration. The correlating numerical and algebraic structural calculations, however, must be separately computed.









1
Photoelastic model of Bourges Cathedral choir. The photoelastic interference patterns are produced by simulated dead weight (static loading).

2
Photoelastic model of Beauvais Cathedral choir. The photoelastic interference patterns are produced by simulated wind loading.

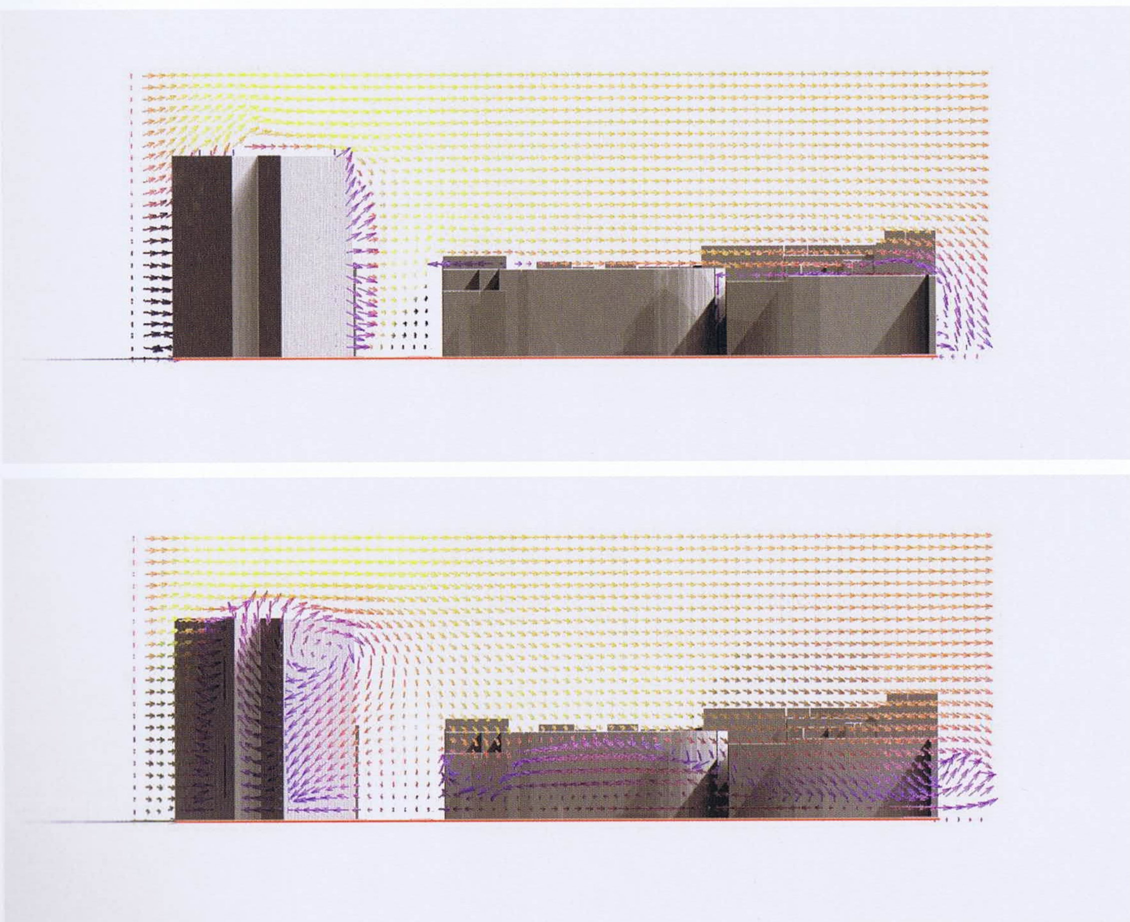
3
Photograph showing how Professor Mark simulated dead weight (static loading) on a model of Beauvais Cathedral using hanging weights of differing masses, attached to corresponding cross-sectional locations.

4
A live loading model of Amiens Cathedral subjected to simulated lateral wind loading. Vertical wires are attached to the model and evenly weighted.

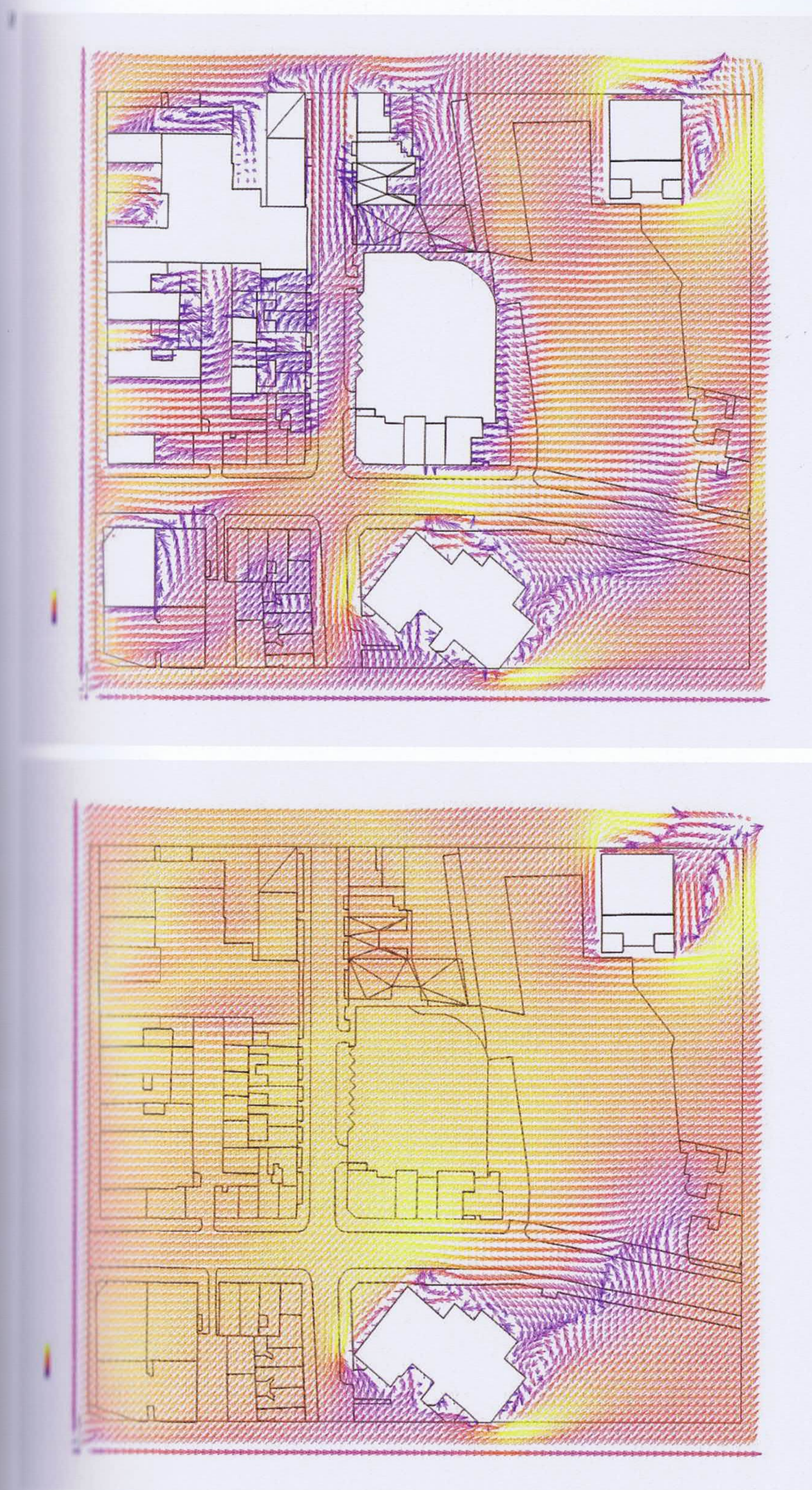
Computational Fluid Dynamics (CFD)

The Navier-Stokes equations, named after Claude-Louis Navier and George Gabriel Stokes, are a set of equations that describe the motion of fluid substances such as liquids and gases. The equations are a dynamical statement of the balance of forces acting at any given region of the fluid. The various numerical approaches to solving the Navier-Stokes equations are collectively called Computational Fluid Dynamics, or CFD. When translated into a graphical format, the motion of the fluids can be seen as particles moving through space. CFD can then be used to simulate wind dynamics—speed and direction—in and around buildings. The architect is able to explore variations in design that can, for example, improve natural ventilation or minimize excessive downdrafts from tall buildings. Using inbuilt or referenced weather data, this analytical computer software allows the user to model and overlay annual wind speed, frequency, and direction, directly on top of a design model, helping the designer develop strategies for natural ventilation, wind shelter, and appropriate structural resistance.

1



1 CFD flow vector analysis section
CFD flow vector analysis showing air movement and velocity in a cross-sectional view of an urban block.

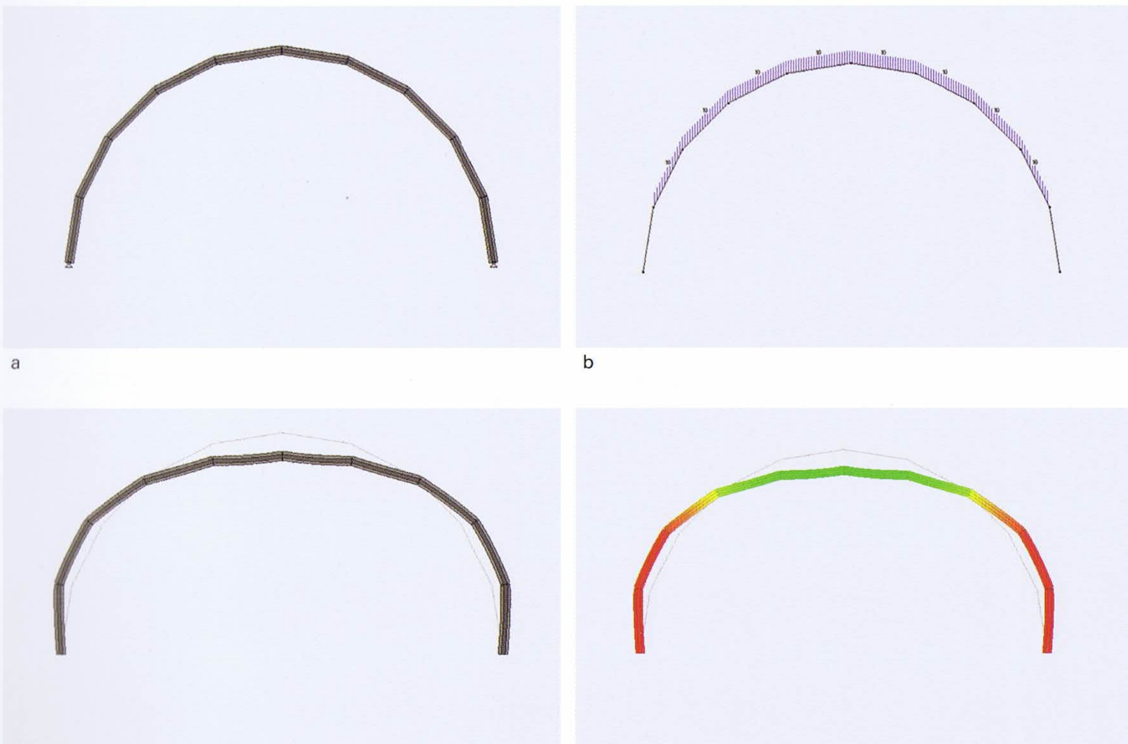


2 CFD flow vector analysis plan
CFD flow vector analysis showing air movement and velocity at two heights above an urban block. Note the prevailing southwesterly wind flow and the turbulence and vortex shedding around the tall building at the center bottom of the images.

Finite Element Analysis (FEA)

The first step in using Finite Element Analysis (FEA) is constructing a finite element model of the structure to be analyzed. Two- or three-dimensional CAD models are imported into FEA software and a "meshing" procedure is used to define and break the model up into a geometric arrangement of small elements and nodes. Nodes represent points at which features such as displacements are calculated. Elements are bounded by sets of nodes and define the localized mass and stiffness properties of the model. Elements are also defined by mesh numbers, which allow reference to be made to corresponding deflections or stresses at specific model locations. Knowing the properties of the materials used, the software then conducts a series of computational procedures to determine effects such as deformations, strains, and stresses, which are caused by applied structural loads. The results can then be studied using visualization tools within the FEA environment to view and to identify the implications of the analysis. Numerical and graphical tools allow the precise location of data such as stresses and deflections to be identified.

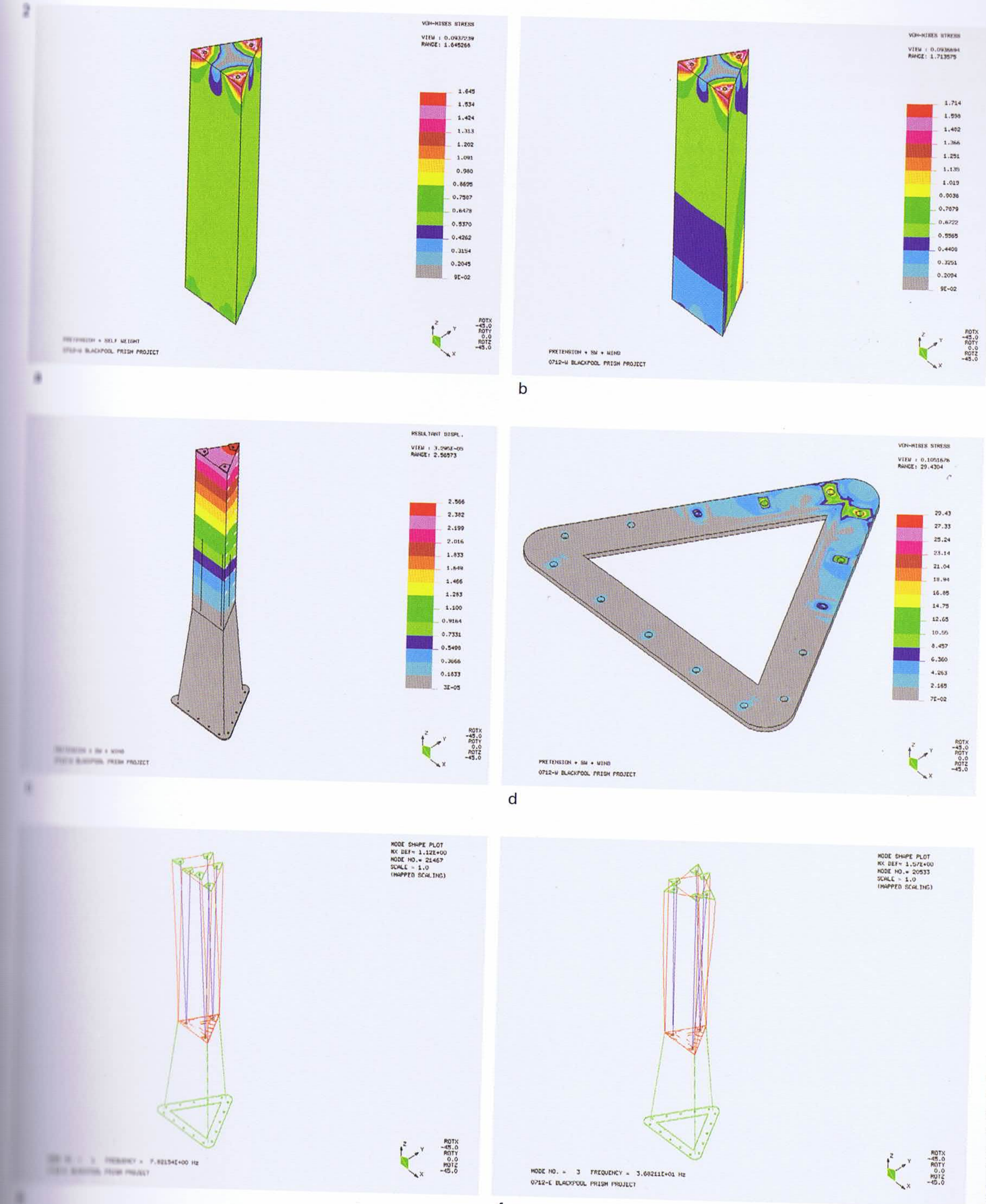
1



a b c d

1 Two-dimensional Finite Element Analysis (FEA)
In the FEA analysis of a simple structure, an arch (a) has a uniform load applied (b). Image c shows how the arch behaves or deforms under load, with sides pushed outward, and the apex lowered. In image d color coding is introduced, representing the internal stress pattern distribution within the arch structure.

2



a b c d e f

2 Acrylic tower project
The following images illustrate the Finite Element Analysis of a 30-foot-tall triangular prismatic tower. The lower 10 feet of the prism comprise a fabricated steel plinth with the remainder manufactured from solid optical-quality acrylic. The prism structure has been analyzed using a three-dimensional computer model and Finite Element Analysis. The structure was modeled using brick elements for the acrylic prism and steel plinth. Steel tensioning rods were used to clamp the acrylic blocks together and were modeled using line elements with temperature boundary conditions applied to produce the desired level of pre-tension. Three models were produced. The first model was to determine the post-tensioning forces and the "along" wind response of the structure; the second model was to determine the "across" (or cross-wind) wind response, and the third model was to determine the effects of temperature on the tension rods. The FEA images present contour plots illustrating the resultant deflections and stress distributions for the "along" wind condition together with the first mode "natural resonant" frequencies and resultant deflections.

Left to right, top to bottom:
a Post-tension induced stress in an acrylic prism around steel rod fixings
b Wind load-induced acrylic stress
c "Along" wind load, showing resultant displacement
d Localized stress in the steel base plate caused by "along" wind load
e Movement caused by "first mode," or natural resonant lateral frequency
f Movement caused by "first mode," or natural resonant torsional frequency